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LETTER TO THE EDITORS

COMMENTS ON THE PAPER "APPLICATION OF PERTURBATION TECHNIQUES TO HEAT-TRANSFER PROBLEMS WITH VARIABLE THERMAL PROPERTIES"

(Received 2 July 1976)

THE ANALYSIS of heat-conduction problems with variable thermal properties, using perturbation techniques presented in [1] deserves due consideration. In the case (b) of above report, that is, transient heat diffusion into a semi-infinite medium with variable heat capacity, the following points are observed and also in cases (a) and (c).

1. In the case (b) of [1] and reference [4] of the above paper, the first order problem is written as $F_1''+2\eta F_1' = 2\eta(F_0-1)F_0'$ omitting the minus sign on RHS, however in the solution presented, this has been taken care of.

2. The numerical value of perturbation parameter $\varepsilon(=\nu T_0)$ is to be verified, for which the coefficients of thermal capacity of metals and alloys can readily be obtained in [3] for various temperature ranges. It is observed that the ε value may take higher value in its limit. It is suggested that non-dimensionalizing ε and F in the following way $\varepsilon = \nu(t_i-t_0)$ and $F = (t_0-t)/(t_0-t_i)$ renders the solution applicable for larger coefficients and wider temperatures, where t_i and t_0 are initial and final temperatures of medium respectively.

Using $C/C_0 = 1+\varepsilon F$, the heat-conduction equation for the case (b) is recast in the following form

$$F''+2\eta F'+\varepsilon 2\eta F = 0$$

the solutions of which are presented up to second order in [4].

3. The first order problem for case (b) is solved using a third-order least square polynomial for erfcη in the range $0 \leqslant \eta \leqslant 3$ replacing the boundary condition $F(\infty) = 0$ by $F(3) = 0$. However a closed form solution is readily obtainable by direct integration maintaining the original boundary conditions. The solutions for first and second order problems are as follows:

$$F_1(\eta) = \left[\frac{1}{\pi}+\frac{1}{\sqrt{\pi}}\eta\,e^{-\eta^2}\right]\operatorname{erf}\eta+\frac{1}{\pi}(e^{-2\eta^2}-1).$$

$$F_2(\eta) = \left[\frac{1}{4\pi}-0.1091-\frac{2\eta}{\pi\sqrt{\pi}}e^{-\eta^2}+\frac{(3+4\eta^2)}{4\pi}e^{-2\eta^2}\right]\operatorname{erf}\eta$$
$$+\frac{\eta}{4\sqrt{\pi}}(1+2\eta^2)e^{-\eta^2}(\operatorname{erf}\eta)^2-\frac{(\operatorname{erf}\eta)^3}{24}$$
$$-\frac{7}{4\pi\sqrt{3}}\operatorname{erf}(\eta\sqrt{3})+\frac{\eta\,e^{-3\eta^2}}{2\pi\sqrt{\pi}}+\frac{\eta\,e^{-\eta^2}}{\pi\sqrt{\pi}}+\frac{(1-e^{-2\eta^2})}{\pi\sqrt{\pi}}.$$

4. The heat-transfer rate at the surface $\eta = 0$, obtained from [1] is compared with the closed form solutions given in (3), in the following table

	$-F'(0)$		
ε	Polynomial for complimentary error *fn*	Direct integration up to I Order	Direct integration up to II Order
0.5	0.9519	0.9486	0.9213
0	1.1280	1.1280	1.1280
−0.5	1.3110	1.3073	1.2800

5. In case (a) the complete first order solution in equation (16) should be

$$\theta = \frac{\sinh y\sin x}{\sinh\pi}+\varepsilon\sum_{n=1,3,\ldots\infty}\frac{2}{n(n^2-4)\pi\sinh^2\pi}$$
$$\times\left[(1-\cosh 2\pi)\frac{\sinh ny}{\sinh n\pi}-(1-\cosh 2y)\right]\sin nx.$$

The second term in the bracket is $\cosh 2Y$ not $\cos 2Y$ as given in equation (6) which can easily be seen from boundary condition $\theta(X,\delta\pi) = \sin X$.

6. In case (c) the term $\cosh Nx\ln\cosh Nx$ is approximated by Mclaurin's series. However the first order problem admits exact solution and using the method of variation of parameters, the solution obtained is:

$$\theta_1(x) = A(x)\cosh Nx+B(x)\sinh Nx$$

where

$$A(x) = A_0-\frac{\operatorname{sech}N}{4}\cosh 2Nx\ln(\operatorname{sech}N\cosh Nx)$$
$$+\frac{\operatorname{sech}N}{4}\cosh^2 Nx-\frac{\operatorname{sech}N}{4}\ln\cosh Nx.$$

$$B(x) = B_0+Nx\frac{\operatorname{sech}N}{2}\ln\operatorname{sech}N$$
$$+\frac{N\operatorname{sech}N}{2}\left[\ln\tfrac{1}{2}+\frac{Nx^2}{2}+\sum_{v=1}^{\infty}\frac{(-1)^v e^{-2vNx}}{2Nv^2}\right]$$
$$+\frac{\operatorname{sech}N}{4}\sinh 2Nx\ln(\operatorname{sech}N\cosh Nx)$$
$$-\frac{N\operatorname{sech}N}{4}\left(\frac{\sinh 2Nx}{2N}-x\right).$$

$$A_0 = \frac{\operatorname{sech}N}{4}\ln\cosh N-\frac{\cosh N}{4}$$
$$-\operatorname{sech}N\tanh N\left[\frac{N}{2}\left(\ln\tfrac{1}{2}+\frac{N}{2}\right)\right.$$
$$\left.+\tfrac{1}{4}\sum_{v=1}^{\infty}\frac{(-1)^v e^{-2Nv}}{v^2}-\frac{\pi^2}{48}-\frac{\sinh 2N}{8}+\frac{N}{4}\right].$$

$$B_0 = \frac{\pi^2}{48}\operatorname{sech}N.$$

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REJOINDER

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THE AUTHORS thank the discussers for providing the closed form solutions for the first and second order problems for case (b) and for the first order problem of case (c). However, for case (c), the second term in the equation for $B(X)$ should read as $\frac{1}{2}NX$ sech N ln sech N cosh NX and not $\frac{1}{2}NX$ sech N ln sech N. Also, in the equation for A_0, the term $\pi^2/48$ should have a plus instead of a minus sign. These can be seen from the boundary conditions, $X = 0$, $\theta_1' = 0$; $X = 1$, $\theta_1 = 0$.

The two minor typographical errors pointed out by the discussers are regretted.

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